Nonlinear realization of local symmetries of AdS space

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Abstract

Coset methods are used to construct the action describing the dynamics associated with the spontaneous breaking of the local symmetries of AdS_{d+1} space due to the embedding of an AdS_d brane. The resulting action is an $SO(2,d)$ invariant AdS form of the Einstein-Hilbert action, which in addition to the AdS_d gravitational vielbein, also includes a massive vector field localized on the brane. Its long wavelength dynamics is the same as a massive Abelian vector field coupled to gravity in AdS_d space.

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1 Introduction

The dynamical degree of freedom of a probe AdS_d brane embedded in an AdS_{d+1} target space is the Nambu-Goldstone world volume field $\phi(x)$, with x^μ the AdS_d world volume coordinates. $\phi(x)$ describes the co-volume oscillatons of the brane in AdS_{d+1} space. Its long wavelength dynamics is given by the reparametrization invariant volume of the brane times the constant brane tension σ and is encoded in an AdS form [1] of the Nambu-Goto action as

$$\begin{aligned}\Gamma &= -\sigma \int d^d x \det e = -\sigma \int d^d x \det \bar{e} \det N \\ &= -\sigma \int d^d x \det \bar{e} \cosh^d \sqrt{m^2\phi^2} \sqrt{1 - \frac{\mathcal{D}_m \phi \eta^{mn} \mathcal{D}_n \phi}{\cosh^2 \sqrt{m^2\phi^2}}}. \end{aligned} \tag{1.1}$$

The induced vielbein $e_\mu{}^m$ has a product form, $e_\mu{}^m = \bar{e}_\mu{}^n N_n{}^m$ where $\bar{e}_\mu{}^m$ is the static background vielbein for the AdS_d world volume of the brane which yields a background world volume Ricci tensor $\bar{R}_{\mu\nu} = m^2(d-1)\bar{g}_{\mu\nu}$ and hence a background Ricci scalar $\bar{R} = m^2 d(d-1)$ with $m^2 > 0$, while the ϕ dependent $N_n{}^m$ is given by

$$\begin{aligned} N_m^{\ n}(x) &= \delta_m^n \cosh\left(\sqrt{m^2\phi^2(x)}\right) \\ &\quad + \left[\sqrt{\left(\cosh^2\left(\sqrt{m^2\phi^2(x)}\right) - \mathcal{D}_r\phi(x)\eta^{rs}\mathcal{D}_s\phi(x)\right)} \right. \\ &\qquad \left. - \cosh\left(\sqrt{m^2\phi^2(x)}\right)\right] \frac{\mathcal{D}_m\phi(x)\mathcal{D}^n\phi(x)}{(\mathcal{D}\phi)^2(x)}, \end{aligned} \tag{1.2}$$

where the derivative $\mathcal{D}_m$ is defined as $\mathcal{D}_m = \bar{e}_m^{-1\mu}\partial_\mu$. Expanding the action through terms bilinear in ϕ gives

$$\Gamma = -\sigma \int d^d x \det \bar{e} \left\{ 1 + \frac{1}{2}(m^2 d)\phi^2 - \frac{1}{2}\partial_\mu\phi \bar{g}^{\mu\nu}\partial_\nu\phi + \cdots \right\}. \tag{1.3}$$

It is seen that the Nambu-Goldstone boson carries the $(E, s) = (d, 0)$ representation [2] of $SO(2, d-1)$. That is, it has mass squared equal to $m^2 d$ and hence energy d in units of m^2 while being spin zero [3]-[4]. It should be noted that the $m^2 = 0$ case reproduces the massless bosonic brane Nambu-Goto action.

The brane action is invariant under a nonlinear realization of the AdS_{d+1} target space global isometry group of transformations $SO(2, d)$. In order to have invariance

under general coordinate transformations, additional gravitational fields must be introduced. The purpose of this paper is to construct the action of the world volume localized gravitational fields when the brane is embedded in curved space [6]-[7]. In short, the dynamics of the oscillating brane in curved space is described by a world volume localized massless graviton represented by a dynamical veilbein $e_\mu{}^m(x)$ and a world volume localized vector field represented by a dynamical field $A_\mu(x)$. As a consequence of the Higgs mechanism [8]-[9], the vector field is massive. The action for these fields is derived in a model independent manner using coset methods in which the AdS_{d+1} local symmetry group $SO(2,d)$ is nonlinearly realized. In section 2, the nonlinear local transformations of the Nambu-Goldstone fields are introduced via the coset method [5]. The locally covariant Maurer-Cartan one-form building blocks for the invariant action are obtained along with the introduction of the dynamical veilbein and vector fields. Derivatives of these Maurer-Cartan world volume vectors that are covariant with respect to local Lorentz and Einstein transformations are defined using the spin and related affine connections. In section 3, these covariant derivatives are used to construct the low energy locally $SO(2,d)$ invariant action. Exploiting the spontaneously broken local (pseudo-) translation and Lorentz transformations, the action is transformed to and analyzed in the unitary gauge. The physical degrees of freedom so obtained are the dynamical world volume veilbein and massive vector field.

2 The Coset Construction

The embedding of AdS_d space spontaneously breaks the symmetry group of the AdS_{d+1} space from $SO(2,d)$ to $SO(2,d-1)$. The low energy action governing the dynamics of the Nambu-Goldstone modes associated with the symmetry breakdown can be constructed using coset methods. This technique begins by introducing the coset element $\Omega \in SO(2,d)/SO(1,d-1)$ where $SO(1,d-1)$ corresponds to the Lorentz structure (stability) group of transformations in AdS_d. The $AdS_d = SO(2,d-1)/SO(1,d-1)$ coordinates, x^μ, act as parameters for pseudo-translations in the world volume and are part of the coset so that

$$\Omega(x) = e^{ix^\mu P_\mu} e^{i\phi(x)Z} e^{iv^\mu(x)K_\mu}, \tag{2.1}$$

The $SO(2,d)$ generators can be expressed in terms of the unbroken $SO(1,d-1)$ Lorentz subgroup representation content of the $SO(2,d-1)$ symmetry group of the brane. The unbroken $SO(2,d-1)$ symmetry group is generated by the subgroup Lorentz transformation generators $M_{\mu\nu}$, where $\mu,\nu = 0,1,2,\ldots,d-1$ and the pseudo-translations in AdS_d space with charges P_μ. The remaining charges are the generating elements of the $SO(2,d)/SO(2,d-1)$ coset. They are the broken $SO(2,d)$ symmetry transformation charges. Z generates the broken $SO(2,d)$ pseudo-translations in the co-volume direction normal to the brane, while K_μ generates the broken AdS_{d+1} Lorentz transformations. Thus the $SO(2,d)$ algebra can be written in terms of the P_μ, $M_{\mu\nu}$, Z and K_μ charges as [1]

$$\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= -i\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\sigma}\right) \\
[M_{\mu\nu}, P_\lambda] &= i\left(P_\mu\eta_{\nu\lambda} - P_\nu\eta_{\mu\lambda}\right) \\
[M_{\mu\nu}, K_\lambda] &= i\left(K_\mu\eta_{\nu\lambda} - K_\nu\eta_{\mu\lambda}\right) \\
[M_{\mu\nu}, Z] &= 0 \\
[P_\mu, P_\nu] &= -im^2 M_{\mu\nu} \\
[K_\mu, K_\nu] &= iM_{\mu\nu} \\
[P_\mu, K_\nu] &= i\eta_{\mu\nu}Z \\
[P_\mu, Z] &= -im^2 K_\mu
\end{aligned}$$

$$[Z, K_\mu] = iP_\mu. \tag{2.2}$$

The coset so defined corresponds to a particular choice of coordinates, specifically denoted as x^μ, for the AdS_d world volume. The fields are also defined as functions of x^μ. The Nambu-Goldstone field $\phi(x)$ along with $v^\mu(x)$ act as the remaining coordinates needed to parametrize the coset manifold $SO(2,d)/SO(2,d-1)$.

Left multiplication of the coset elements Ω by an $SO(2,d)$ group element g which is specified by local infinitesimal parameters $\epsilon^\mu(x), z(x), b^\mu(x), \lambda^{\mu\nu}(x)$ so that

$$g(x) = e^{i\epsilon^\mu(x)P_\mu} e^{iz(x)Z} e^{ib^\mu(x)K_\mu} e^{\frac{i}{2}\lambda^{\mu\nu}(x)M_{\mu\nu}}, \tag{2.3}$$

results in transformations of the space-time coordinates and the Nambu-Goldstone fields according to the general form [5]

$$g(x)\Omega(x) = \Omega'(x')h(x). \tag{2.4}$$

The transformed coset element, Ω', is a function of the transformed world volume coordinates and the total variations of the fields so that

$$\Omega' = e^{ix'^\mu P_\mu} e^{i\phi'(x')Z} e^{iv'^\mu(x')K_\mu}, \tag{2.5}$$

while h is a field dependent element of the stability group $SO(1,d-1)$:

$$h = e^{\frac{i}{2}\theta^{\mu\nu}(x)M_{\mu\nu}}. \tag{2.6}$$

Exploiting the algebra of the $SO(2,d)$ charges displayed in equation (2.2), along with extensive use of the Baker-Campbell-Hausdorf formulae, the local $SO(2,d)$ transformations are obtained as

$$\begin{aligned}
x'^\mu &= \left[1 - z(x)\sqrt{m^2}\tanh\sqrt{m^2\phi^2}\frac{\sin\sqrt{4m^4x^2}}{\sqrt{m^2x^2}}\right]x^\mu - \lambda^{\mu\nu}(x)x_\nu \\
&\quad + \left[P_L^{\mu\nu}(x) + \sqrt{m^2x^2}\cot\sqrt{m^2x^2}P_T^{\mu\nu}(x)\right]\epsilon_\nu(x) \\
&\quad + \frac{\tanh\sqrt{m^2\phi^2}}{\sqrt{m^2}}\left[\cos\sqrt{m^2x^2}P_L^{\mu\nu}(x) + \frac{\sqrt{m^2x^2}}{\sin\sqrt{m^2x^2}}P_T^{\mu\nu}(x)\right]b_\nu(x) \\
\phi'(x') &= \phi(x) + z(x)\cos\sqrt{m^2x^2} + b_\mu(x)x^\mu\frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
v'^{\mu}(x') &= v^{\mu}(x) - \lambda^{\mu\nu}(x) v_{\nu} - \frac{m^2}{2} \frac{\tan\sqrt{m^2 x^2/4}}{\sqrt{m^2 x^2/4}} (\epsilon^{\mu}(x) x^{\nu} - \epsilon^{\nu}(x) x^{\mu}) v_{\nu} \\
&\qquad - z(x) \frac{m^2}{\cosh\sqrt{m^2\phi^2}} \frac{\sin\sqrt{m^2 x^2}}{\sqrt{m^2 x^2}} \left[P_L^{\mu\nu}(v) + \sqrt{v^2}\coth\sqrt{v^2} P_T^{\mu\nu}(v) \right] x_{\nu} \\
&\qquad + \sqrt{m^2} \frac{\tan\sqrt{m^2 x^2/4}}{\sqrt{m^2 x^2}} \tanh\sqrt{m^2\phi^2} (b^{\mu}(x) x^{\nu} - b^{\nu}(x) x^{\mu}) v_{\nu} \\
&\quad + \frac{1}{\cosh\sqrt{m^2\phi^2}} \left[P_L^{\mu\nu}(v) + \sqrt{v^2}\coth\sqrt{v^2} P_T^{\mu\nu}(v) \right] \\
&\qquad\qquad \times \left[\cos\sqrt{m^2 x^2} P_{L\nu\rho}(x) + P_{T\nu\rho}(x) \right] b^{\rho}(x)
\end{aligned}
$$

$$
\begin{aligned}
\theta^{\mu\nu}(x) &= \lambda^{\mu\nu}(x) + \frac{m^2}{2} \frac{\tan\sqrt{m^2 x^2/4}}{\sqrt{m^2 x^2/4}} (\epsilon^{\mu}(x) x^{\nu} - \epsilon^{\nu}(x) x^{\mu}) \\
&\qquad - z(x) \frac{m^2}{\cosh\sqrt{m^2\phi^2}} \frac{\sin\sqrt{m^2 x^2}}{\sqrt{m^2 x^2}} (v^{\mu} x^{\nu} - v^{\nu} x^{\mu}) \frac{\tanh\sqrt{v^2/2}}{\sqrt{v^2}} \\
&\qquad - \sqrt{m^2} \frac{\tan\sqrt{m^2 x^2/4}}{\sqrt{m^2 x^2}} \tanh\sqrt{m^2\phi^2} (b^{\mu}(x) x^{\nu} - b^{\nu}(x) x^{\mu}) \\
&\quad - \frac{1}{\cosh\sqrt{m^2\phi^2}} \frac{\tanh\sqrt{v^2/2}}{\sqrt{v^2}} \\
&\qquad\quad \times \left[\cos\sqrt{m^2 x^2} P_L^{\mu\rho}(x) b_{\rho}(x) v^{\nu} + P_T^{\mu\rho}(x) b_{\rho}(x) v^{\nu} - (\mu \leftrightarrow \nu) \right].
\end{aligned}
\tag{2.7}
$$

Here the transverse and longitudinal projectors for x^{μ} are defined as

$$
\begin{aligned}
P_{T\mu\nu}(x) &= \eta_{\mu\nu} - \frac{x_{\mu} x_{\nu}}{x^2} \\
P_{L\mu\nu}(x) &= \frac{x_{\mu} x_{\nu}}{x^2}
\end{aligned}
\tag{2.8}
$$

and $\eta_{\mu\nu}$ is the metric tensor for d–dimensional Minkowski space having signature $(+1, -1, \ldots, -1)$. In the above, the indices are raised, lowered and contracted using $\eta_{\mu\nu}$. Both Nambu-Goldstone fields ϕ and v^{μ} transform inhomogeneously under the broken local translations Z and broken local Lorentz transformations K_{μ}. Thus these broken transformations can be used to transform to the unitary gauge in which both ϕ and v^{μ} vanish. This will be done in section 3 in order to exhibit the physical degrees of freedom in a more transparent fashion.

The nonlinearly realized $SO(2,d)$ transformations induce a coordinate and field

dependent general coordinate transformation of the world volume space-time coordinates. From the x^μcoordinate transformation given above, the AdS_{d+1} general coordinate Einstein transformation for the world volume space-time coordinate differentials is given by

$$dx'^\mu = dx^\nu G_\nu{}^\mu(x), \tag{2.9}$$

with $G_\nu{}^\mu(x) = \partial x'^\mu/\partial x^\nu$. The $SO(2,d)$ invariant interval can be formed using the metric tensor $g_{\mu\nu}(x)$ so that $ds^2 = dx^\mu g_{\mu\nu}(x)dx^\nu = ds'^2 = dx'^\mu g'_{\mu\nu}(x')dx'^\nu$ where the metric tensor transforms as

$$g'_{\mu\nu}(x') = G^{-1\rho}_\mu(x) g_{\rho\sigma}(x) G^{-1\sigma}_\nu(x). \tag{2.10}$$

The form of the vielbein (and hence the metric tensor) as well as the locally $SO(2,d)$ covariant derivatives of the Nambu-Goldstone boson fields and the spin connection can be extracted from the locally covariant Maurer-Cartan one-form, $\Omega^{-1}D\Omega$, which can be expanded in terms of the generators as

$$\begin{aligned} \Omega^{-1}D\Omega &\equiv \Omega^{-1}(d+i\hat{E})\Omega \\ &= i\left[\omega^m P_m + \omega_Z Z + \omega_K^m K_m + \frac{1}{2}\omega_M^{mn} M_{mn}\right]. \end{aligned} \tag{2.11}$$

Here Latin indices $m,n = 0,1,\ldots,d-1$, are used to distinguish tangent space local Lorentz transformation properties from world volume Einstein transformation properties which are denoted using Greek indices. In what follows Latin indices are raised and lowered with use of the Minkowski metric tensors, η^{mn} and η_{mn}, while Greek indices are raised and lowered with use of the curved AdS_d metric tensors, $g^{\mu\nu}$ and $g_{\mu\nu}$. Since the Nambu-Goldstone fields vanish in the unitary gauge it is useful to exhibit the one-form gravitational fields in terms of their pseudo-translated form

$$\hat{E} = e^{+ix^\mu P_\mu} E e^{-ix^\mu P_\mu}. \tag{2.12}$$

The world volume one-form gravitational fields E have the expansion in terms of the charges as

$$E = E^m P_m + AZ + B^m K_m + \frac{1}{2}\gamma^{mn} M_{mn}. \tag{2.13}$$

Similarly expanding $\hat{E}$ as

$$\hat{E} = \hat{E}^m P_m + \hat{A} Z + \hat{B}^m K_m + \frac{1}{2}\hat{\gamma}^{mn} M_{mn}, \tag{2.14}$$

one finds the various fields are related according to

$$\begin{aligned}
\hat{E} &= \left[\cos\sqrt{m^2x^2} P^m_{Tn}(x) + P^m_{Ln}(x)\right] E^n - \frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}}\gamma^{mn} x_n \\
\hat{A} &= A\cos\sqrt{m^2x^2} - \frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}} B^m x_m \\
\hat{B^m} &= \left[P^m_{Tn}(x) + \cos\sqrt{m^2x^2} P^m_{Ln}(x)\right] B^n + m^2\frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}} A x^m \\
\hat{\gamma}^{mn} &= \gamma^{mn} - m^2\frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}}\left(E^m x^n - E^n x^m\right) \\
&\quad - \left(\cos\sqrt{m^2x^2} - 1\right)\left[\gamma^{ms} P^n_{Ls}(x) - \gamma^{ns} P^m_{Ls}(x)\right].
\end{aligned} \tag{2.15}$$

Defining the one-form gravitational fields to transform as a gauge field so that

$$\hat{E}'(x') = g(x)\hat{E}(x)g^{-1}(x) - ig(x)dg^{-1}(x), \tag{2.16}$$

the covariant Maurer-Cartan one-form transforms analogously to the way it varied for global transformations:

$$\omega'(x') = h(x)\omega(x)h^{-1}(x) + h(x)dh^{-1}(x), \tag{2.17}$$

with $h = e^{\frac{i}{2}\theta^{mn}(x)M_{mn}}$. Expanding in terms of the $SO(2,d)$ charges, the individual one-forms transform according to their local Lorentz nature

$$\begin{aligned}
\omega'^m(x') &= \omega^n(x)\Lambda_n{}^m(\theta(x)) \\
\omega'_Z(x') &= \omega_Z(x) \\
\omega'^m_K(x') &= \omega^n_K(x)\Lambda_n{}^m(\theta(x)) \\
\omega'^{mn}_M(x') &= \omega^{rs}_M(x)\Lambda_r{}^m(\theta(x))\Lambda_s{}^n(\theta(x)) - d\theta^{mn}(x).
\end{aligned} \tag{2.18}$$

For infinitesimal transformations, the local Lorentz transformations are $\Lambda_n{}^m(\theta(x)) = \delta_n{}^m + \theta_n{}^m(x)$, while the infinitesimal local $SO(2,d)$ transformations of the gravitational one-forms take the form

$$\hat{E}'^m = \hat{E}^m + \hat{\gamma}^{mn}\epsilon_n - z\hat{B}^m + b^m\hat{A} - \lambda^{mn}\hat{E}_n - d\epsilon^m$$

$$
\begin{aligned}
\hat{A}' &= \hat{A} - \epsilon_m \hat{B}^m + b_m \hat{E}^m - dz \\
\hat{B}'^m &= \hat{B}^m + \epsilon^m m^2 \hat{A} - z m^2 \hat{E}^m + \hat{\gamma}^{mn} b_n - \lambda^{mn} \hat{B}_n - db^m \\
\hat{\gamma}'^{mn} &= \hat{\gamma}^{mn} + m^2(\epsilon^m \hat{E}^n - \epsilon^n \hat{E}^m) - (b^m \hat{B}^n - b^n \hat{B}^m) \\
&\qquad + (\lambda^{mr} \hat{\gamma}^n{}_r - \lambda^{nr} \hat{\gamma}^m{}_r) - d\lambda^{mn}.
\end{aligned} \tag{2.19}
$$

Using the Feynman formula for the variation of an exponential operator in conjunction with the Baker-Campell-Hausdorff formulae, the individual world volume one-forms appearing in the above decomposition of the covariant Maurer-Cartan one-form are secured as

$$
\begin{aligned}
\omega^m &= dx^\mu e_\mu{}^m \\
&= dx^\mu \mathcal{E}_\mu{}^n N_n{}^m \\
\omega_Z &= dx^\mu \omega_{Z\mu} \\
&= dx^\mu \cosh\sqrt{v^2}\, \mathcal{E}_\mu{}^m \left[-v_m \frac{\tanh\sqrt{v^2}}{\sqrt{v^2}} \cosh\sqrt{m^2\phi^2} + \mathcal{E}_m^{-1\nu} \left(\partial_\nu \phi + A_\nu\right) \right] \\
\omega_K^m &= dx^\mu \omega_{K\mu}^m \\
&= \left[P_L^{mn}(v) + \frac{\sinh\sqrt{v^2}}{\sqrt{v^2}} P_T^{mn}(v) \right] (dv_n - (\bar{\omega}_{Mnr} + \gamma_{nr}) v^r) \\
&\quad + \left[P_L^{mn}(v) + \cosh\sqrt{v^2}\, P_T^{mn}(v) \right] \left[(\bar{\omega}_n + E_n)\sqrt{m^2} \sinh\sqrt{m^2\phi^2} \right. \\
&\qquad\qquad \left. + B_n \cosh\sqrt{m^2\phi^2} \right] \\
\omega_M^{mn} &= dx^\mu \omega_{M\mu}^{mn} \\
&= (\bar{\omega}_M^{mn} + \gamma^{mn}) - \cosh\sqrt{m^2\phi^2} \frac{\sinh\sqrt{v^2}}{\sqrt{v^2}} \left[B^m v^n - B^n v^m \right] \\
&\quad + \sqrt{m^2} \frac{\sinh\sqrt{v^2}}{\sqrt{v^2}} \sinh\sqrt{m^2\phi^2} \left[v^m P_{Ts}^n(v) - v^n P_{Ts}^m(v) \right] (\bar{\omega}^s + E^s) \\
&\quad + \left[\cosh\sqrt{v^2} - 1 \right] \left[\frac{v^m dv^n - v^n dv^m}{v^2} \right. \\
&\qquad\qquad \left. - \left(P_{Lr}^m(v)(\bar{\omega}_M^{nr} + \gamma^{nr}) - P_{Lr}^n(v)(\bar{\omega}_M^{mr} + \gamma^{mr}) \right) \right].
\end{aligned} \tag{2.20}
$$

In these expressions, the background AdS_d covariant coordinate differential, $\bar{\omega}^m$, and spin connection, $\bar{\omega}_M^{mn}$, are obtained from the AdS_d coordinate one-form $(e^{-ix^m P_m})d(e^{ix^n P_n}) = i[\bar{\omega}^m P_m + \frac{1}{2}\bar{\omega}_M^{mn} M_{mn}]$ as

$$
\bar{\omega}^m = \frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}} P_T^{mn}(x) dx_n + P_L^{mn}(x) dx_n
$$

$$\bar{\omega}_M^{mn} \;=\; \left[\cos\sqrt{m^2x^2}-1\right]\frac{(x^m dx^n - x^n dx^m)}{x^2}. \tag{2.21}$$

The background differential $\bar{\omega}^m$ is related to the x^μ world volume coordinate differential by the background veilbein $\bar{e}_\mu{}^m(x)$ as

$$\bar{\omega}^m = dx^\mu \bar{e}_\mu{}^m(x). \tag{2.22}$$

Using equation (2.21) along with $d = dx^\mu \partial_\mu^x$, the background veilbein is obtained as

$$\bar{e}_\mu{}^m(x) = \frac{\sin\sqrt{m^2x^2}}{\sqrt{m^2x^2}} P_{T\mu}{}^m(x) + P_{L\mu}{}^m(x). \tag{2.23}$$

The covariant coordinate differential ω^m is related to the world volume coordinate differential dx^μ by the vielbein $e_\mu{}^m$ which in turn can be written in a factorized form as the product of the dynamic vielbein $\mathcal{E}_\mu{}^m$ and the Nambu-Goto vielbein $N_n{}^m$

$$\begin{aligned}
\mathcal{E}_\mu{}^m &= \bar{e}_\mu{}^m + E_\mu{}^m + B_\mu{}^m \frac{\tanh\sqrt{m^2\phi^2}}{\sqrt{m^2}} \\
N_n{}^m &= \cosh\sqrt{m^2\phi^2}\left[P_{Tn}{}^m(v) + \cosh\sqrt{v^2}P_{Ln}{}^m(v)\right] \\
&\qquad - \cosh\sqrt{v^2}\,\mathcal{E}_n^{-1\nu}(\partial_\nu\phi + A_\nu)v^m \frac{\tanh\sqrt{v^2}}{\sqrt{v^2}}.
\end{aligned} \tag{2.24}$$

The one-forms and their covariant derivatives are the building blocks of the locally $SO(2,d)$ invariant action. Indeed a m^{th}-rank contravariant local Lorentz and n^{th}-rank covariant Einstein tensor, $T_{\mu_1\cdots\mu_n}^{m_1\cdots m_m}$ is defined to transform as [10]

$$T'^{m'_1\cdots m'_m}_{\mu'_1\cdots\mu'_n}(x') = G^{-1\mu_1}_{\mu'_1}(x)\cdots G^{-1\mu_n}_{\mu'_n}(x)T^{m_1\cdots m_m}_{\mu_1\cdots\mu_n}(x)\Lambda_{m_1}^{\;m'_1}(\theta(x))\cdots\Lambda_{m_m}^{\;m'_m}(\theta(x)). \tag{2.25}$$

For example, the veilbein transforms as $e'^m_\mu(x') = G^{-1\nu}_\mu(x)e_\nu{}^n(x)\Lambda_n{}^m(\theta(x))$. Hence, the veilbein and its inverse can be used to convert local Lorentz indices into world volume indices and vice versa. Since the Minkowski metric, η_{mn}, is invariant under local Lorentz transformations the metric tensor $g_{\mu\nu}$

$$g_{\mu\nu} = e_\mu{}^m \eta_{mn} e_\nu{}^n, \tag{2.26}$$

is a rank 2 Einstein tensor. It can be used to define covariant Einstein tensors given contravariant ones. Likewise, the Minkowski metric can be used to define covariant local Lorentz tensors given contravariant ones.

Since the Jacobian of the $x^\mu \to x'^\mu$ transformation is simply

$$d^dx' \;\; = \;\; d^dx \;\; \det G, \tag{2.27}$$

it follows that $d^dx' \;\; \det e'(x') = d^dx \;\; \det e(x)$ since $\det \Lambda = 1$. Thus an $SO(2,d)$ invariant action is constructed as

$$\Gamma = \int d^dx \det e(x) \mathcal{L}(x), \tag{2.28}$$

with the Lagrangian an invariant $\mathcal{L}'(x') = \mathcal{L}(x)$. The invariants that make up the Lagrangian can be found by contracting the indices of tensors with the appropriate vielbein, its inverse and the Minkowski metric. For example $\omega_{Z\mu} g^{\mu\nu} \omega_{Z\nu}$ is an invariant term used to construct the action.

Besides products of the covariant Maurer-Cartan one-forms, their covariant derivatives can also be used to construct invariant terms of the Lagrangian. The covariant derivative of a general tensor can be defined using the affine and related spin connections. Consider the covariant derivative of the Lorentz tensor T^{mn}

$$\nabla_\rho T^{mn} = \partial_\rho T^{mn} - \omega^m_{M\rho r} T^{rn} - \omega^n_{M\rho r} T^{mr}. \tag{2.29}$$

Since the spin connection transforms inhomogeneously according to equation (2.18), the covariant derivative of T^{mn} transforms homogeneously again

$$(\nabla_\rho T^{mn})' = G^{-1\sigma}_\rho (\nabla_\sigma T^{rs}) \Lambda_r{}^m \Lambda_s{}^n. \tag{2.30}$$

Converting the Lorentz index n to a world index ν using the vielbein, the covariant derivative for mixed tensors is obtained

$$\nabla_\rho T^{m\nu} \;\; \equiv \;\; e^{-1\nu}_n \nabla_\rho T^{mn} = \partial_\rho T^{m\nu} - \omega^{mr}_{M\rho} T_r{}^\nu + \Gamma^\nu_{\sigma\rho} T^{m\sigma}, \tag{2.31}$$

where the spin connection $\omega^{mn}_{M\rho}$ and $\Gamma^\nu_{\sigma\rho}$ are related according to [10]

$$\Gamma^\nu_{\sigma\rho} = e^{-1\nu}_n \partial_\rho e_\sigma{}^n - e^{-1\nu}_n \omega^{nr}_{M\rho} e_\sigma{}^s \eta_{rs}. \tag{2.32}$$

(Note that this relation as well follows from the requirement that the covariant derivative of the vielbein vanishes, $\nabla_\rho e_\mu{}^m = 0$.) Applying the above to the Minkowski metric

Lorentz 2-tensor yields the formula relating the affine connection $\Gamma^{\rho}_{\mu\nu}$ to derivatives of the metric

$$\begin{aligned}
\nabla_\rho \eta^{mn} &= \partial_\rho \eta^{mn} - \omega^{m}_{M\rho r}\eta^{rn} - \omega^{n}_{M\rho r}\eta^{mr} = -\omega^{mn}_{M\rho} - \omega^{nm}_{M\rho} \\
&= 0 \\
&= e_{\mu}{}^{m} e_{\nu}{}^{n} \nabla_\rho g^{\mu\nu} \\
&= e_{\mu}{}^{m} e_{\nu}{}^{n} \left(\partial_\rho g^{\mu\nu} + \Gamma^{\mu}_{\sigma\rho} g^{\sigma\nu} + \Gamma^{\nu}_{\sigma\rho} g^{\mu\sigma} \right). \qquad (2.33)
\end{aligned}$$

The solution to this equation yields the affine connection in terms of the derivative of the metric [10] (the space is torsionless, hence the connection is symmetric $\Gamma^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\nu\mu}$)

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left[\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right]. \qquad (2.34)$$

Finally a covariant field strength two-form can be constructed out of the inhomogeneously transforming spin connection $\omega^{mn}_{M\mu}$

$$F^{mn} = d\omega^{mn}_{M} + \eta_{rs} \omega^{mr}_{M} \wedge \omega^{ns}_{M}. \qquad (2.35)$$

Expanding the forms yields the field strength tensor

$$F^{mn}_{\mu\nu} = \partial_\mu \omega^{mn}_{M\nu} - \partial_\nu \omega^{mn}_{M\mu} + \eta_{rs} \omega^{mr}_{M\mu} \omega^{ns}_{M\nu} - \eta_{rs} \omega^{mr}_{M\nu} \omega^{ns}_{M\mu}. \qquad (2.36)$$

It can be shown that $F^{mn}_{\mu\nu} = e^{-1n\sigma} e_{\rho}{}^{m} R^{\rho}{}_{\sigma\mu\nu}$ where $R^{\rho}{}_{\sigma\mu\nu}$ is the Riemann curvature tensor

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\nu \Gamma^{\rho}_{\sigma\mu} - \partial_\mu \Gamma^{\rho}_{\sigma\nu} + \Gamma^{\lambda}_{\sigma\mu} \Gamma^{\rho}_{\lambda\nu} - \Gamma^{\lambda}_{\sigma\nu} \Gamma^{\rho}_{\lambda\mu}. \qquad (2.37)$$

The Ricci tensor is given by $R_{\mu\nu} = R^{\rho}_{\mu\nu\rho}$ and hence the scalar curvature is an invariant

$$R = g^{\mu\nu} R_{\mu\nu} = -e^{-1\mu}_{m} e^{-1\nu}_{n} F^{mn}_{\mu\nu}. \qquad (2.38)$$

3 The Invariant Action

The covariant derivatives of the Maurer-Cartan one-forms provide additional building blocks out of which the invariant action is to be constructed. For example the covariant derivatives of $\omega_{Z\nu}$ and $\omega_{K\nu}^n$ yield the mixed tensors

$$\begin{aligned}\nabla_\mu \omega_{Z\nu} &= \partial_\mu \omega_{Z\nu} - \Gamma^\rho_{\mu\nu}\omega_{Z\rho} \\ \nabla_\mu \omega_{K\nu}^n &= \partial_\mu \omega_{K\nu}^n - \Gamma^\rho_{\mu\nu}\omega^n_{K\rho} - \omega^{nr}_{M\mu}\omega^s_{K\nu}\eta_{rs}.\end{aligned} \tag{3.1}$$

So proceeding, the invariant action describing the curved AdS_d brane embedded in curved AdS_{d+1} space has the general low energy form

$$\begin{aligned}\Gamma = \int d^dx \det e \Big\{ & \Lambda + \kappa^2 R + \frac{1}{2}\omega_{Z\mu}\left[(M^2+\xi R)g^{\mu\nu} + \zeta R^{\mu\nu}\right]\omega_{Z\nu} \\ & -\frac{1}{2}\nabla_\mu\omega_{Z\nu}\nabla_\rho\omega_{Z\sigma}\left[Z_1(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + Z_2 g^{\mu\nu}g^{\rho\sigma}\right] \\ & +\frac{1}{2}\omega^m_{K\mu}\omega^n_{K\nu}\left[a e_m^{-1\mu}e_n^{-1\nu} + b e_n^{-1\mu}e_m^{-1\nu} + c g^{\mu\nu}\eta_{mn}\right] \\ & -\omega^m_{K\mu}\nabla_\nu\omega_{Z\rho}\left[\alpha e_m^{-1\mu}g^{\nu\rho} + \beta e_m^{-1\nu}g^{\mu\rho} + \gamma e_m^{-1\rho}g^{\mu\nu}\right]\Big\}.\end{aligned} \tag{3.2}$$

Many invariant terms are possible. The above includes a reduced set of terms which leads to a consistent effective theory. The model can be further simplified by setting the parameters ξ and ζ to zero. On the other hand, due to the Higgs mechanism, the parameter M cannot be zero and is an independent scale in the theory. Since the massive vector A_μ is a Proca field, it can be consistently quantized by further setting Z_2 to zero. Moreover, exploiting the identity $\partial_\mu(\det e\, T^\mu) = \det e\, \nabla_\mu T^\mu$ along with the chain rule for covariant differentiation, integration by parts has been used to eliminate redundant terms. A term of the form $\det e e_m^{-1\mu}\omega^m_{K\mu}$ has also been excluded from the action since, when $\omega^m_{K\mu}$ is eliminated as below, it will not result in any new terms.

The action is independent of any terms containing derivatives acting on $\omega^m_{K\mu}$. Hence varying the action with respect to $\omega^m_{K\mu}$ yields an algebraic identity relating it

to $\nabla_\mu \omega_{Z\nu}$ as

$$
\begin{aligned}
& \omega_{K\mu}^{m}\left[a e_{m}^{-1\mu} e_{\sigma}^{\ r} + b\delta_{\sigma}^{\mu}\delta_{m}^{r} + c e^{-1\mu r} e_{\sigma m}\right] \\
= \ & \nabla_\nu \omega_{Z\rho}\left[\alpha g^{\nu\rho} e_{\sigma}^{\ r} + \beta e^{-1\rho r}\delta_{\sigma}^{\nu} + \gamma e^{-1\nu r}\delta_{\sigma}^{\rho}\right].
\end{aligned}
\tag{3.3}
$$

Introducing $\omega_{K\mu}^{m} = e^{-1\nu m} B_{\mu\nu}$ allows the solution

$$
\begin{aligned}
B_{\sigma\rho} &= g_{\rho\sigma}\frac{1}{(b+c)}\left[\alpha - a\frac{(\alpha d + \beta + \gamma)}{(ad + b + c)}\right] g^{\mu\nu}\nabla_\mu \omega_{Z\nu} \\
&\quad + \frac{(\beta+\gamma)}{2(b+c)}\left[\nabla_\sigma \omega_{Z\rho} + \nabla_\rho \omega_{Z\sigma}\right] + \frac{(\beta-\gamma)}{2(b-c)}\left[\nabla_\sigma \omega_{Z\rho} - \nabla_\rho \omega_{Z\sigma}\right].
\end{aligned}
\tag{3.4}
$$

Substituting this back into the action allows $\omega_{K\mu}^{m}$ to be eliminated in favor of terms involving the vielbein and $\omega_{Z\mu}$ yielding

$$
\begin{aligned}
\Gamma &= \int d^d x \det e \left\{ \Lambda + \kappa^2 R + \frac{1}{2}\omega_{Z\mu}\left[(M^2 + \xi R) g^{\mu\nu} + \zeta R^{\mu\nu}\right]\omega_{Z\nu} \right. \\
&\qquad \left. - \frac{1}{2} Z_1 \nabla_\mu \omega_{Z\nu} \nabla_\rho \omega_{Z\sigma}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \right\},
\end{aligned}
\tag{3.5}
$$

where the form of the contractions of the product of two $\nabla_\mu \omega_{Z\nu}$ terms are similar to those of the initial action and hence the constants have just been redefined and the effective Z_2 has been set to zero. Exploiting the form of the covariant derivative of the Z one-form in order to define the anti-symmetric field strength tensor $F_{\mu\nu}$,

$$
F_{\mu\nu} = (\nabla_\mu \omega_{Z\nu} - \nabla_\nu \omega_{Z\mu}) = (\partial_\mu \omega_{Z\nu} - \partial_\nu \omega_{Z\mu}),
\tag{3.6}
$$

the action becomes

$$
\begin{aligned}
\Gamma &= \int d^d x \det e \left\{ \Lambda + \kappa^2 R + \frac{1}{2}\omega_{Z\mu}\left[(M^2 + \xi R) g^{\mu\nu} + \zeta R^{\mu\nu}\right]\omega_{Z\nu} \right. \\
&\qquad \left. - \frac{Z_1}{4} F_{\mu\nu} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \right\}.
\end{aligned}
\tag{3.7}
$$

According to equation (2.7), ϕ and v^m transform inhomogeneously under the broken translation and Lorentz transformation local transformations. Hence we now fix the unitary gauge defined by $\phi = 0 = v^m$. So doing, the covariant one-forms take a simplified form

$$
\omega^m = dx^\mu e_\mu^{\ m} = dx^\mu \mathcal{E}_\mu^{\ m} = dx^\mu (\bar{e}_\mu^{\ m} + E_\mu^{\ m})
$$

$$\begin{aligned}
\omega_Z &= dx^\mu A_\mu \\
\omega_K^m &= dx^\mu B_\mu{}^m \\
\omega_M^{mn} &= (\bar{\omega}_M^{mn} + \gamma^{mn}) = dx^\mu (\bar{\omega}_{M\mu}^{mn} + \gamma_\mu^{mn}).
\end{aligned} \tag{3.8}$$

Note that, in this gauge, equation (2.24) reduces to $\mathcal{E}_\mu{}^m = \bar{e}_\mu{}^m + E_\mu{}^m$ and $N_b{}^a = \delta_b{}^a$. Consequently the vielbein $e_\mu{}^m = \mathcal{E}_\mu{}^b N_b{}^a = \bar{e}_\mu{}^m + E_\mu{}^m$ and thus depends only on the gravitational fluctuation field, $E_\mu{}^m$, about the AdS_d background vielbein $\bar{e}_\mu{}^m$ and is independent of the vector field. As such, the $\det e$ gives no contribution to the vector mass even though it is the source of Nambu-Goldstone boson kinetic term in the model with spontaneously broken global isometry. Instead, the mass of the vector, M, is a completely new scale arising from an independent monomial. This realization of the Higgs mechanism is strikingly different from what occurs when gauging internal symmetries. In that case, when the symmetry is made local, the Nambu-Goldstone boson kinetic term gets replaced by the square of the covariant derivative containing the vector connection. In unitary gauge, the Nambu-Goldstone field vanishes leaving the residual vector mass term whose scale is set by the Nambu-Goldstone decay constant, a scale already present in the global model.

The action, equation (3.7), reduces to that of a massive vector field coupled to a gravitational field with cosmological constant

$$\Gamma = \int d^d x \det e \left\{ \Lambda + \kappa^2 R - \frac{Z_1}{4} F_{\mu\nu} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} + \frac{1}{2} A_\mu \left[(M^2 + \xi R) g^{\mu\nu} + \zeta R^{\mu\nu} \right] A_\nu \right\}, \tag{3.9}$$

with the field strength tensor $F_{\mu\nu}$ for the vector field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3.10}$$

The action was constructed by considering gravitational fluctuations about a background static AdS space and describes the embedding of a brane in that curved space. The world volume action of the brane is equivalent to that of a world volume gravitational field Einstein-Hilbert action, with corresponding cosmological constant as dictated by the field equations evaluated on the AdS background, and the action for a massive vector field in that gravitating space. Furthermore, the action can equally

well be used to describe a bosonic brane embedded in a space gravitating about a background Minkowski space by taking the limit $m^2 \to 0$. The vector field remains massive with the mass M still being an independent scale. Setting the parameters ξ and ζ to zero, the world volume action for a brane embedded in curved space has the form of a massive Abelian gauge theory coupled to gravity.

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